INSTITUT DES HAUTES ÉTUDES

POUR LE DÉVELOPPEMENT DE LA CULTURE, DE LA SCIENCE ET DE LA TECHNOLOGIE EN BULGARIE

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Concours Général de Physiques

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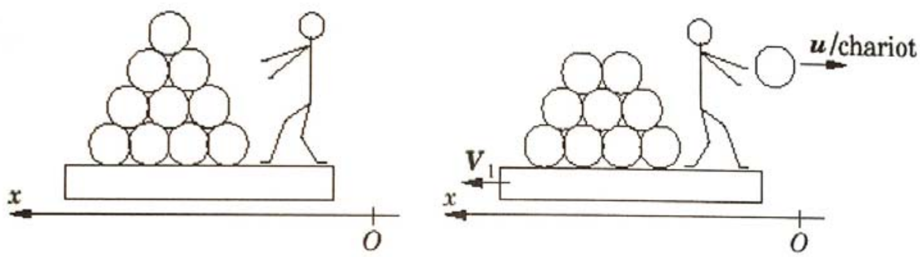
The main problem and the two exercises are completely independent and can be solved in any order.
All the answers must be given in English or French. The clarity and precision will be taken into account for the final note.
The exam is 4 hours long! The calculators are authorized.

Problem: Some aspects of the astronautics (12pts)

The main goal of this problem is to obtain numerical values. An analytical expression of the result as a function of the given parameters should be given, but it's just a bypass to the final solution.

I – Propulsion by reaction.

In this part we will study the principle of propulsion. To this end, we will consider a board with wheels or a trolley on which we have a machinist and ***n*** sacks filled with sand and of individual masse ***m*** everyone. We neglect all dissipative effects. In order to simplify the expressions, we will also neglect the masse of the trolley and the machinist itself compared to the masse of every sand-bag. The referential system ***R***, attached to the ***Ox*** axis is galilean.



I.A At time $t = 0$, the operator throw out the first sack of masse m, with speed $\mathbf{u} = -u\mathbf{e}_x$ * compared to the speed of the board! Show that, in the referential R attached to the ground (to the Ox axis), the impulsion of one system (which you should define precisely) is

* Every symbol in bold should be considered as a vector variable.

conserved. Using this general property, find the speed $\mathbf{V}_1$ (evaluated in R) of the trolley and all that is on it, after this first throw.

<u>I.B</u> At time $t_2 = t_1 + T$, the machinist throw out a second sack, with speed $\mathbf{u} = -u\mathbf{e}_x$ compared to the speed of the board! Evaluate the speed (in R) of the trolley and all that is on it, after this second throw. Show that it could be rewritten as $\mathbf{V}_2 = -\left(\frac{1}{n-1} + \frac{1}{n-2}\right)\mathbf{u}$. Define the studied system.

<u>I.C</u> Establish the expression of the speed $\mathbf{V}_k$ of the trolley, always evaluated in the referential R, after the k-th throw effectuated at time $t_1 + (k-1)T$, as a function of n, k and $\mathbf{u}$. Define the studied system.

<u>I.D</u> Establish the expression $\mathbf{a}_k$ of the average acceleration of the trolley during a period of time including the k-th throw (for instance, between $t_1 + (k-3/2)T$ and $t_1 + (k-1/2)T$) as a function of n, k, T and $\mathbf{u}$ (in R).

<u>I.E</u> We call D_m the "outflow of masse" or "masse debit", which is the masse thrown out of the trolley per time unit. Express $\mathbf{a}_k$ as a function of n, k, D_m, m and $\mathbf{u}$.

<u>I.F</u> Show that the system {trolley and all that it contains after the k-th throw} seems to be exposed, during a period of time including the throw, to a pushing force Π average that you should express as a function of D_m and $\mathbf{u}$.

<u>II – Propulsion by a rocket engine.</u>

Now, we will study of a total masse (at the moment t) $m(t)$ and speed $\mathbf{V}(t)$ in a galilean referential R ; we suppose that the "outflow of masse" D_m of the effected gas is constant and that the speed of ejection is $\mathbf{u}$ in the referential R' attached to the rocket. All the external forces are represented by one force noted $\mathbf{R}$.

<u>II.A</u> By making a detailed balance of the impulsion of one closed system between the time t and $t + dt$, show that $m(t)\frac{d\mathbf{V}(t)}{dt} = \mathbf{R} + \mathbf{T}$, where $\mathbf{T}$ is a "pushing force", which expression you should precise by means of $\mathbf{u}$ and D_m.

<u>II.B</u> We suppose that the rocket is moving in the empty space, we neglect all gravitational forces; the initial and final masse of the rocket is respectively m_i and m_f ; $\mathbf{u}$ and $\mathbf{V}(t)$ have the same fixed direction. Give the increase of the speed $\Delta V = V_f - V_i$ as a function of m_i, m_f and u where u is the norm of $\mathbf{u}$ supposed constant.

<u>II.C</u> We define the propulsion efficiency Q as the ratio of the kinetic energy, communicated to the "useful masse" m_f of a rocket accelerated from a stand still position, to the total energy that is used, defined as $m_e u^2 / 2$, where m_e is the ejected mass between the initial and the final moment. Give the expression of Q as a function of $x = V_f / u$. Draw Q as a function of x. How can you explain qualitatively the existence of a maximum?

II.D The total initial masse of the rocket Saturn V was $2\times10^6\,\mathrm{kg}$, the speed of ejection of the gases was $4\ \mathrm{km.s^{-1}}$ and the take-off acceleration about 1g ($g \cong 10\ \mathrm{m.s^{-2}}$). Evaluate the "masse outflow" and discuss.

In all the preceding questions the referential R ($Oxyz$) is attached to the earth.

II.E At $t = 0$, a rocket without initial speed, situated at $z = 0$ is ignited. We suppose that the rocket is ascending vertically in a gravitational field $\mathbf{g}$ (supposed constant), with a "masse outflow" D_m, also supposed constant. The planet is without atmosphere. Find the expression giving the evolution of the speed $V(t)$ and the altitude $z(t)$ as a function of $m(0), g, u$ and D_m. We remind you that $\int \ln x dx = x\ln(x) - x$.

II.F The radius of the Earth is $R_T = 6400\ \mathrm{km}$. The intensity of the Earth's gravity on the surface is g_0 ($\mathrm{g}_0 \cong 10\ \mathrm{m.s^{-2}}$). Evaluate the potential energy of a masse of 1kg. at rest on the Earth surface, supposing its potential energy at infinity as zero.

II.G The energy produced by the combustion of one kilogram of dioxygen and dihydrogen mixture is less than $2\times10^7\,\mathrm{J}$. In these conditions, is it possible to overcome the gravitation field of the Earth? We ask for a qualitative analysis.

III. – The ionic engine.

The ejection of matter could be done not only by a chemical, but also by an electrical way.

III.A Inside a spacecraft, charged particles of masse μ and positive charge q are accelerated by a potential U_t and after that ejected outside the spacecraft. Suppose that D_m is the ejected "masse outflow". Express the "pushing force" T as a function of q, U_t, D_m and μ.

III.B In the case, where we neglect the gravitational force and the air resistance, evaluate the minimal electric power $P_{e\min}$ that the engine should supply, so that the acceleration of the spacecraft is γ. $P_{e\min}$ should be expressed as a function of γ, μ, q, U_t and the total instant masse m of the spacecraft at the.

III.C Numerical applications: The engine NSTAR in the probe Deep Space One, launched on 24 October 1998, had the following characteristics:

- Accelerating tension: $U_t = 1090\mathrm{V}$
- Consumed electrical power by the engine: $2{,}3\mathrm{kW}$
- Ejected particles: ions Xenon, charge $+e$ ($N_A = 6{,}02\times10^{23}\,\mathrm{mol^{-1}}$, the molar masse of Xenon is $M_{Xe} = 131{,}3\mathrm{g.mol^{-1}}$, the elementary electrical charge is $e = 1{,}6\times10^{-19}\,\mathrm{C}$)
- Pushing force: $93\mathrm{mN}$

Calculate the ratio $\mathrm{T/P_e}$ in two different ways. Is the provided data consistent? (an answer limited to "Yes" or "No" will be considered insufficient)

Exercise 1 (3pts)

A plano-concave lens having index of refraction $n = 1{,}50$ is placed on a flat glass plate, as shown in ***Figure 1***. Its curved surface, with radius of curvature 8.00 m, is on the bottom. The lens is illuminated from above with yellow sodium light of wavelength 589 nm, and a series of concentric bright and dark rings is observed by reflection. The interference pattern has a dark spot at the center, surrounded by 50 dark rings, of which the largest is at the outer edge of the lens.

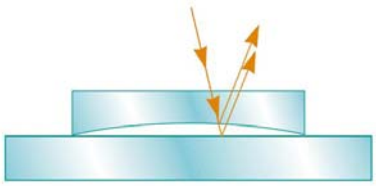


Figure 1

(a) What is the thickness of the air layer at the center of the interference pattern?
(b) Calculate the radius of the outermost dark ring.
(c) Find the focal length of the lens.

Exercise 2 (5pts)

Consider a perfectly conducting disk of radius r_0 in a constant magnetic field B perpendicular to the plane of the disk. Sliding contacts are provided at the edge of the disk (C_1) and at its axle (C_2) (cf. ***Figure 2***). This system is Faraday's "homopolar generator." When turned at constant angular velocity, it provides a large direct current with no ripple. A torque is produced by a mass M hung on a long string wrapped around the perimeter of the disk.

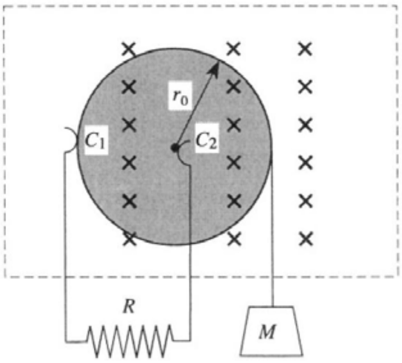


Figure 2

(a) Explain. how and why a current flows. Give a quantitative expression for the current as a function of angular velocity.
(b) Given a long enough string, this system will reach a constant angular velocity ω_f. Find this ω_f and the associated current.

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